

Lesson 39: Improper Fractions and Mixed Numbers



You have been adding, subtracting, multiplying and dividing proper fractions. We can do the same operations with an improper fraction. An **improper fraction** is a fraction where the numerator value (top number) is larger than the denominator (bottom number).

Examples are: $\frac{5}{2}, \frac{8}{7}, \frac{3}{2}$



You are able to convert improper fractions back to the proper form by dividing the denominator into the numerator.

Example: $\frac{5}{2} = 2\frac{1}{2}$ \leftarrow whole number Your remainder is then placed over the divisor
 $\frac{4}{1}$ \leftarrow remainder

Answer: $\frac{5}{2} = 2\frac{1}{2}$



Now you are working with a mixed number ($2\frac{1}{2}$).

A **mixed number** (sometimes referred to as a mixed fraction) has both a whole number and a fraction.

This illustration shows you five halves:

| | | |
|---------------|---------------|---------------|
| $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $\frac{1}{2}$ | $\frac{1}{2}$ | |

$$= \frac{5}{2}$$

This illustration shows you five halves: You can see how:

| | | | | | | | | |
|---------------|---|---------------|---|---------------|---|--|--------------------------------------|-----------------|
| $\frac{1}{2}$ | + | $\frac{1}{2}$ | = | $\frac{2}{2}$ | → | $\frac{2}{2} = 1 \text{ whole}$ | → | 1 |
| $\frac{1}{2}$ | + | $\frac{1}{2}$ | = | $\frac{2}{2}$ | → | + $\frac{2}{2} = 1 \text{ whole}$ | → | +1 |
| $\frac{1}{2}$ | | | = | $\frac{1}{2}$ | → | + $\frac{1}{2} = \frac{1}{2} \text{ fraction}$ | → | + $\frac{1}{2}$ |
| | | | | $\frac{5}{2}$ | = | | $2 \frac{1}{2} \text{ mixed number}$ | |

Now it is time for you to try converting improper fractions to mixed numbers.

Take Lesson 39 Quiz 1



Now you can take a mixed fraction and convert it to an improper fraction. Remember our discussion where....

$$\frac{5}{2} = \begin{array}{|c|c|c|} \hline \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \hline \frac{1}{2} & \frac{1}{2} & \\ \hline \end{array}$$

$$1 \text{ whole} + 1 \text{ whole} + \frac{1}{2} = 2 \frac{1}{2}$$

To convert a mixed number, you need to break it down to see how many parts you can make from the mixed number. In $2 \frac{1}{2}$, you are working with halves. 2 wholes = how many halves?

$$\begin{array}{|c|c|} \hline \frac{1}{2} & \frac{1}{2} \\ \hline \frac{1}{2} & \frac{1}{2} \\ \hline \end{array} = \frac{4}{2} \text{ halves}$$

Then add the fraction

$$\begin{array}{r} + \frac{1}{2} \\ \hline \frac{5}{2} \end{array}$$



There is a trick to learn to do this step easily. In the mixed number $2\frac{1}{2}$, multiply the whole number by the denominator, then add the numerator.

Example: Step 1) Multiply 2 (whole number) x 2 (denominator) = 4

Step 2) Add + 1 (mixed number numerator) $\underline{+ 1}$

You now have the numerator of the improper fraction $\rightarrow 5$

Step 3) Bring the denominator over, making it $\frac{5}{2}$

Let's change some mixed numbers to improper fractions!

Example 1: $3\frac{1}{2} = \frac{7}{2}$; ($3 \times 2 = 6 + 1 = 7$)

Example 2: $7\frac{3}{8} = \frac{59}{8}$; ($7 \times 8 = 56 + 3 = 59$)

Now, let's change the improper fraction back to a mixed number!

Example 1:

$$\frac{7}{2} \rightarrow \begin{array}{r} 2 \overline{) 7} \\ \underline{6} \\ 1 \end{array} \rightarrow 3\frac{1}{2} \quad \left| \quad \begin{array}{r} 2 \overline{) 3} \\ \underline{2} \\ 1 \end{array} \rightarrow 1\frac{1}{2} \quad \left| \quad \begin{array}{r} 2 \overline{) 7} \\ \underline{6} \\ 1 \end{array} \rightarrow 3\frac{1}{2}$$

Example 2:

$$\frac{59}{8} \rightarrow \begin{array}{r} 8 \overline{) 59} \\ \underline{56} \\ 3 \end{array} \rightarrow 7\frac{3}{8}$$

Take Lesson 39 Quiz 2



You will need to make these conversions in order to add, subtract, multiply or divide fractions. See the following --

Add:

$$\begin{array}{r} 2\frac{5}{8} \\ + \frac{4}{8} \\ \hline 2\frac{9}{8} \end{array}$$

Since $\frac{9}{8}$ is improper, we can convert by dividing the denominator into the numerator and get a mixed number. Then, add to the whole number.

$$\begin{array}{r} 1 \\ 8 \overline{)9} = 1\frac{1}{8} \\ \underline{8} \\ 1 \end{array}$$

Answer:

$$\begin{array}{r} 2\frac{9}{8} = 1\frac{1}{8} \\ + 2 \\ \hline 3\frac{1}{8} \end{array} \text{ This is your final answer in correct form.}$$

Let's try multiplication!

Multiply:

$$2\frac{1}{2} \times 1\frac{1}{8}$$

Both are mixed numbers. You need to convert both to improper fractions in order to carry out the operation.

$$2\frac{1}{2} = \frac{2 \times 2 + 1}{2} = \frac{5}{2}$$

$$1\frac{1}{8} = \frac{1 \times 8 + 1}{8} = \frac{9}{8}$$

Now multiply:

$$\frac{5}{2} \times \frac{9}{8} = \frac{45}{16}$$

Change the improper fraction above to a mixed number.

$$16 \overline{)45} = 2\frac{13}{16}$$

(The answer is complete. No need to reduce the fraction.)

Now you try!

Take Lesson 39 Quiz 3



Try this – Divide mixed numbers!

Did you remember all the steps?

Step 1 - convert mixed numbers

Step 2 – invert (flip) the second fraction and multiply

Step 3 - convert the improper fraction back to a mixed number in lowest terms

$$1\frac{1}{2} \div 1\frac{1}{6} \longrightarrow \frac{3}{2} \div \frac{7}{6} \longrightarrow \frac{3}{2} \times \frac{6}{7} = \frac{18}{14} \longrightarrow 1\frac{4}{14} \longrightarrow 1\frac{2}{7}$$

Great! Now you try to solve some problems.

Take Lesson 39 Quiz 4