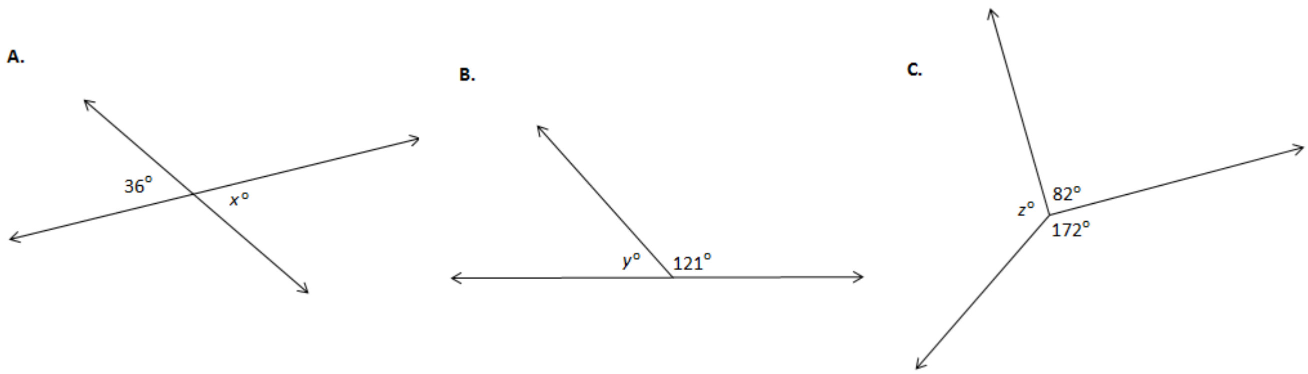
Lesson 6: Solve for Unknown Angles—Angles and Lines at a Point

Classwork

Opening Exercise

Determine the measure of the missing angle in each diagram.

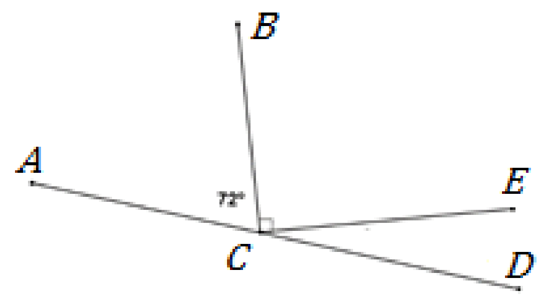


What facts about angles did you use?

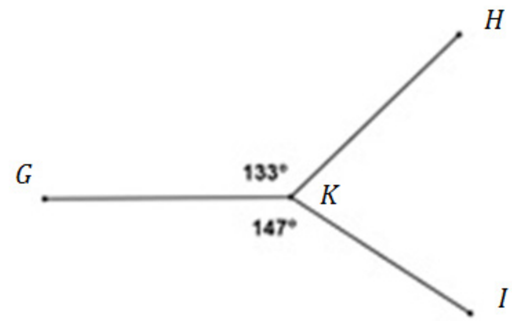
Discussion

Two angles $\angle AOC$ and $\angle COB$, with a common side $\overrightarrow{OC}$, are __________________ if C belongs to the interior of $\angle AOB$. The sum of angles on a straight line is $180°$, and two such angles are called a *linear pair*. Two angles are called *supplementary* if the sum of their measures is __________________; two angles are called *complementary* if the sum of their measures is __________________. Describing angles as supplementary or complementary refers only to the measures of their angles. The positions of the angles or whether the pair of angles is adjacent to each other is not part of the definition.

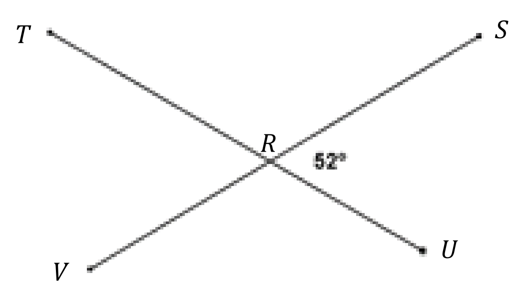
In the figure, line segment AD is drawn.
Find $m\angle DCE$.



The total measure of adjacent angles around a point is ______.
Find the measure of $\angle HKI$.

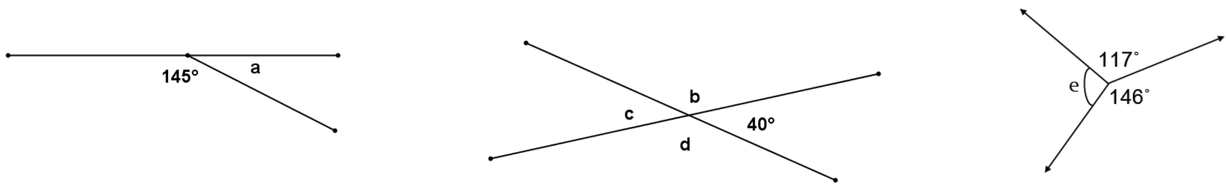


Vertical angles have __________ measure. Two angles are vertical if their sides form opposite rays.
Find $m\angle TRV$.



Example

Find the measures of each labeled angle. Give a reason for your solution.

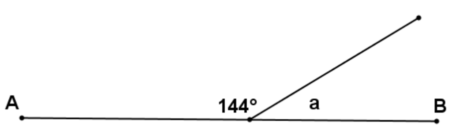


Angle	Angle Measure	Reason
$\angle a$		
$\angle b$		
$\angle c$		
$\angle d$		
$\angle e$		

Exercises

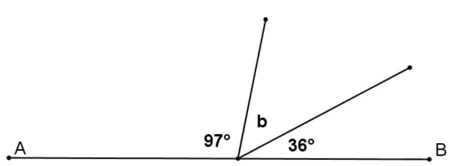
In the figures below, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are straight line segments. Find the measure of each marked angle, or find the unknown numbers labeled by the variables in the diagrams. Give reasons for your calculations. Show all the steps to your solutions.

1.



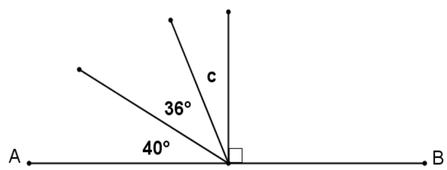
$m\angle a =$ ____________________

2.



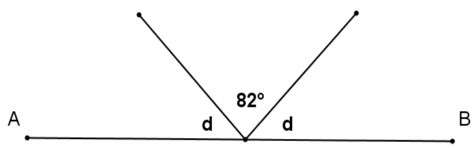
$m\angle b =$ ____________________

3.



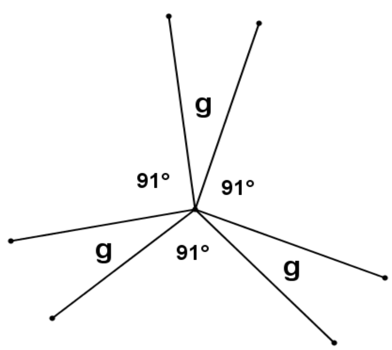
$m\angle c =$ ____________________

4.



$m\angle d =$ ____________________

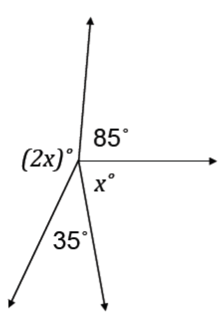
5.



$m\angle g =$ ____________________

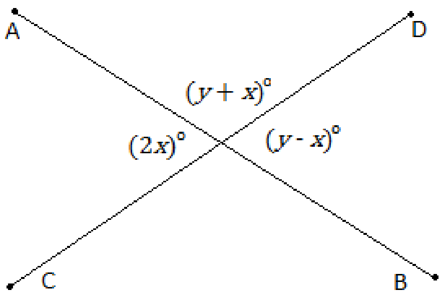
For Exercises 6–12, find the values of x and y. Show all work.

6.



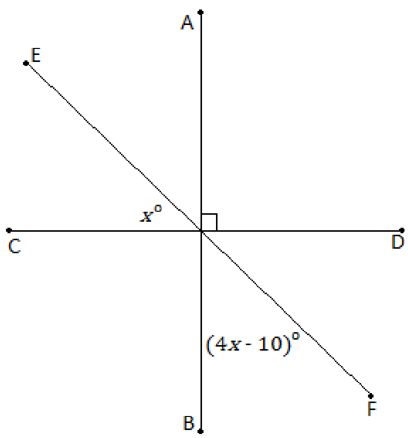
$x =$ ____________________

7.


A
D
$(y+x)^\circ$
$(2x)^\circ$
$(y-x)^\circ$
C
B

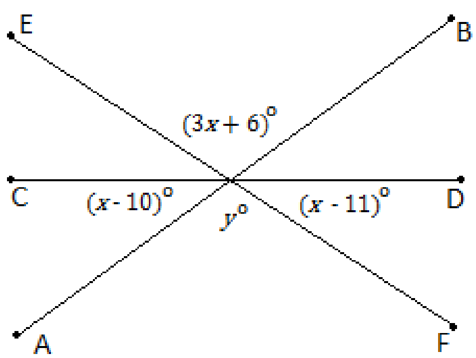
x = ______________ y = ___________

8.


A
E
x°
C
D
$(4x-10)^\circ$
B
F

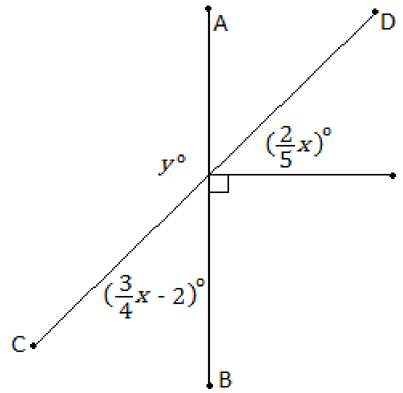
x = ___________________

9.


E
B
$(3x+6)^\circ$
C
$(x-10)^\circ$
$(x-11)^\circ$
D
y°
A
F

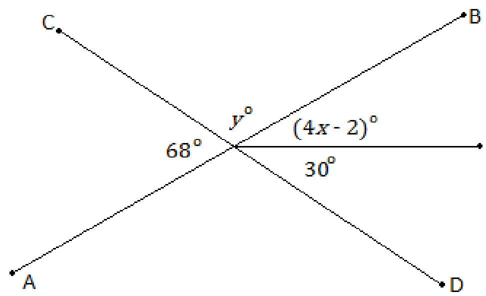
x = ______________ y = ___________

10.



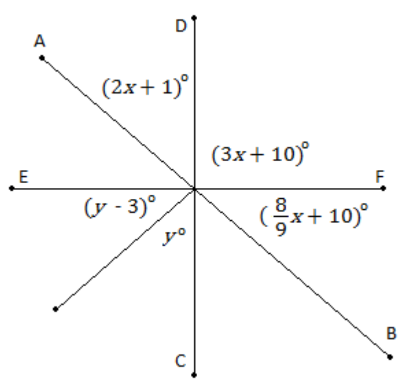
$x =$ ____________ $y =$ ____________

11.



$x =$ ____________ $y =$ ____________

12.



$x =$ ____________ $y =$ ____________

Relevant Vocabulary

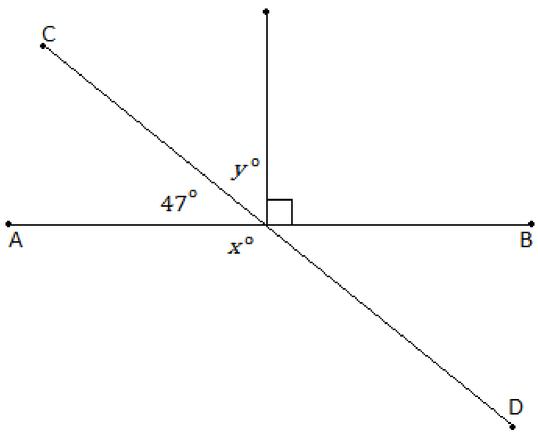
STRAIGHT ANGLE**:** If two rays with the same vertex are distinct and collinear, then the rays form a line called a *straight angle*.

VERTICAL ANGLES**:** Two angles are *vertical angles* (or vertically opposite angles) if their sides form two pairs of opposite rays.

Problem Set

In the figures below, $\overline{AB}$ and $\overline{CD}$ are straight line segments. Find the value of x and/or y in each diagram below. Show all the steps to your solutions, and give reasons for your calculations.

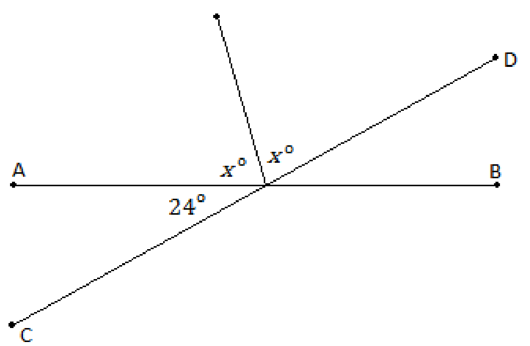
1.



x = ____________________

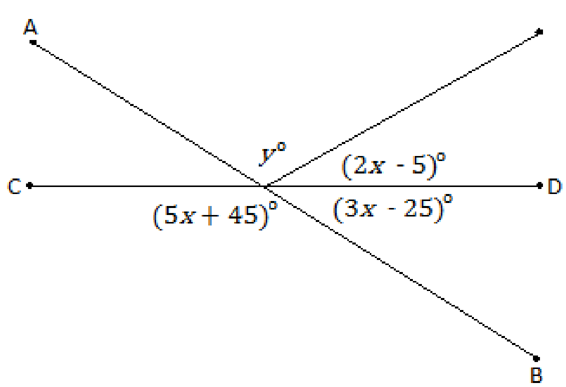
y = ____________________

2.



x = ____________________

3.



x = ____________________

y = ____________________

Lesson 7: Solve for Unknown Angles—Transversals

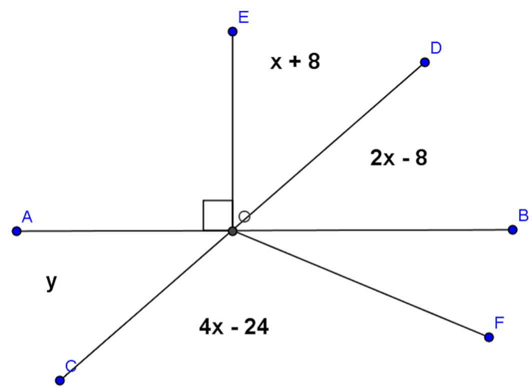
Classwork

Opening Exercise

Use the diagram at the right to determine x and y.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are straight lines.

$x =$ ________

$y =$ ________



Name a pair of vertical angles:

____

Find the measure of $\angle BOF$. Justify your calculation.

____

____

Discussion

Given line AB and line CD in a plane (see the diagram below), a third line EF is called a *transversal* if it intersects $\overleftrightarrow{AB}$ at a single point and intersects $\overleftrightarrow{CD}$ at a single but different point. Line AB and line CD are parallel if and only if the following types of angle pairs are congruent or supplementary.

- Corresponding angles are equal in measure.

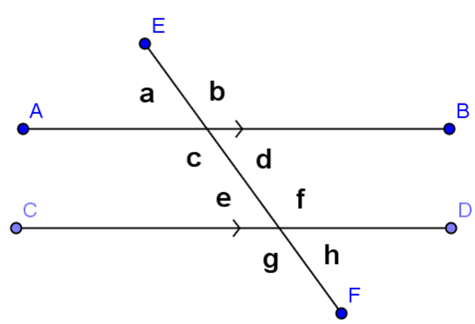
  ____

- Alternate interior angles are equal in measure.

  ____

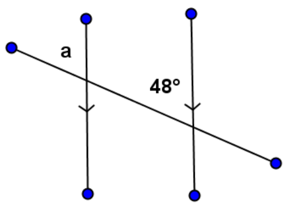
- Same-side interior angles are supplementary.

  ____



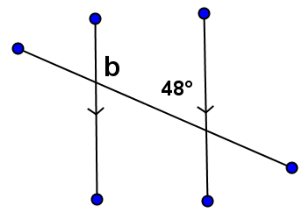
Examples

1.



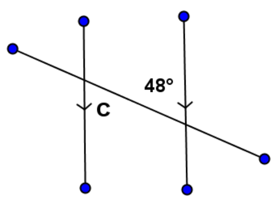
$m\angle a =$ ________

2.



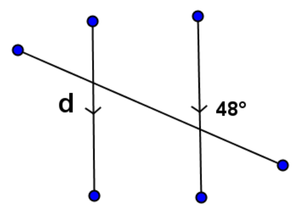
$m\angle b =$ ________

3.



$m\angle c =$ ________

4.

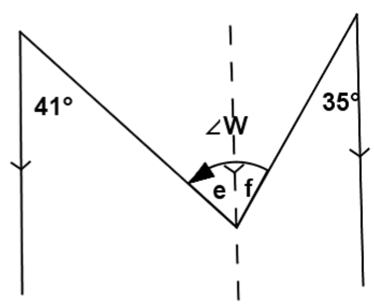


$m\angle d =$ ________

5. An ________________________________ is sometimes useful when solving for unknown angles.

   In this figure, we can use the auxiliary line to find the measures of $\angle e$ and $\angle f$ (how?) and then add the two measures together to find the measure of $\angle W$.

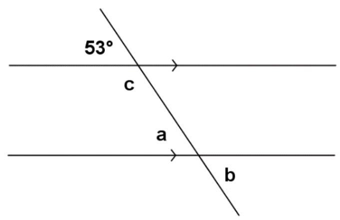
   What is the measure of $\angle W$?



Exercises 1–10

In each exercise below, find the unknown (labeled) angles. Give reasons for your solutions.

1.

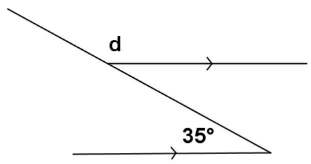


$m\angle a$ = ____________________

$m\angle b$ = ____________________

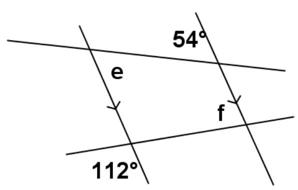
$m\angle c$ = ____________________

2.



$m\angle d$ = ____________________

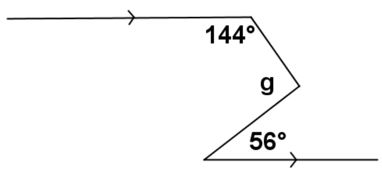
3.



$m\angle e$ = ____________________

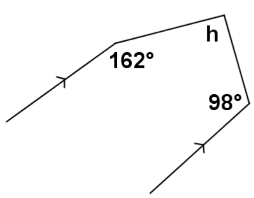
$m\angle f$ = ____________________

4.



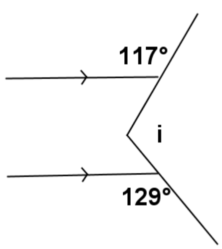
$m\angle g$ = ____________________

5.



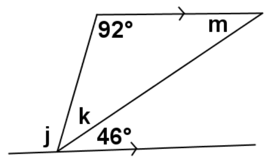
$m\angle h$ = ____________________

6.



$m\angle i$ = ____________________

7.

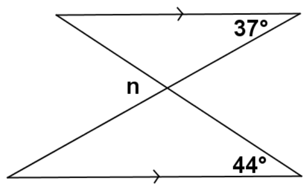


$m\angle j =$ ______________________

$m\angle k =$ ______________________

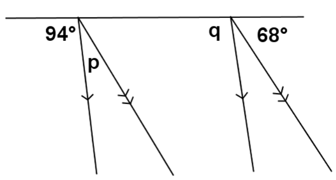
$m\angle m=$ ______________________

8.



$m\angle n =$ ______________________

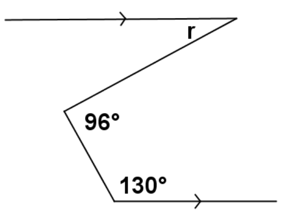
9.



$m\angle p =$ ______________________

$m\angle q =$ ______________________

10.



$m\angle r =$ ______________________

Relevant Vocabulary

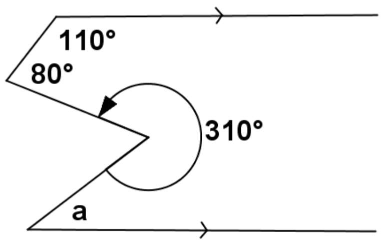
ALTERNATE INTERIOR ANGLES: Let line t be a transversal to lines l and m such that t intersects l at point P and intersects m at point Q. Let R be a point on line l and S be a point on line m such that the points R and S lie in opposite half-planes of t. Then $\angle RPQ$ and $\angle PQS$ are called *alternate interior angles* of the transversal t with respect to line m and line l.

CORRESPONDING ANGLES*:* Let line t be a transversal to lines l and m. If $\angle x$ and $\angle y$ are alternate interior angles and $\angle y$ and $\angle z$ are vertical angles, then $\angle x$ and $\angle z$ are *corresponding angles*.

Problem Set

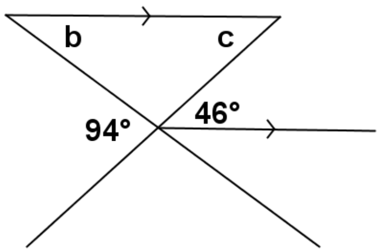
Find the unknown (labeled) angles. Give reasons for your solutions.

1.



$m\angle a =$ ____________________

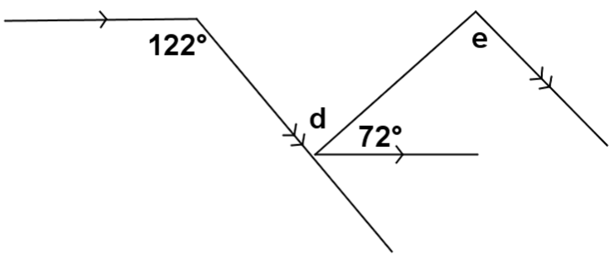
2.



$m\angle b =$ ____________________

$m\angle c =$ ____________________

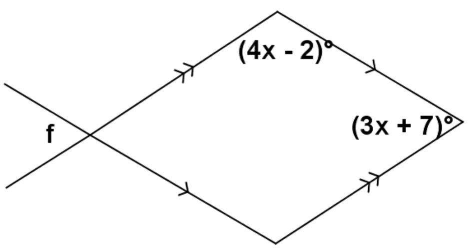
3.



$m\angle d =$ ____________________

$m\angle e =$ ____________________

4.



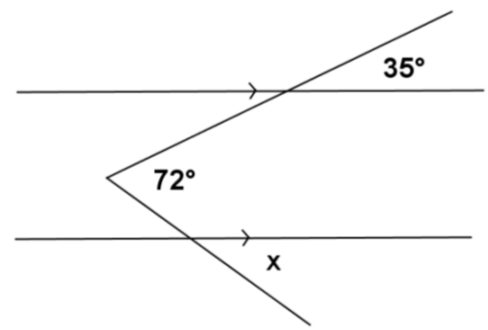
$m\angle f =$ ____________________

Lesson 8: Solve for Unknown Angles—Angles in a Triangle

Classwork

Opening Exercise

Find the measure of angle x in the figure to the right. Explain your calculations. (Hint: Draw an auxiliary line segment.)



Discussion

The sum of the 3 angle measures of any triangle is ________________.

INTERIOR OF A TRIANGLE: A point lies in the *interior of a triangle* if it lies in the interior of each of the angles of the triangle.

In any triangle, the measure of the exterior angle is equal to the sum of the measures of the ______________ angles. These are sometimes also known as ________________ angles.

Base angles of an ______________ triangle are equal in measure.

Each angle of an ______________ triangle has a measure equal to $60°$.

Relevant Vocabulary

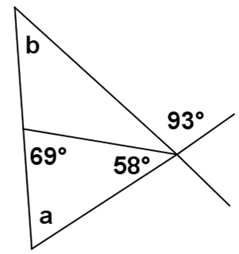
ISOSCELES TRIANGLE: An *isosceles triangle* is a triangle with at least two sides of equal length.

ANGLES OF A TRIANGLE: Every triangle $\triangle ABC$ determines three angles, namely, $\angle BAC$, $\angle ABC$, and $\angle ACB$. These are called the *angles of* $\triangle ABC$.

EXTERIOR ANGLE OF A TRIANGLE: Let $\angle ABC$ be an interior angle of a triangle $\triangle ABC$, and let D be a point on $\overleftrightarrow{AB}$ such that B is between A and D. Then $\angle CBD$ is an *exterior angle of the triangle* $\triangle ABC$.

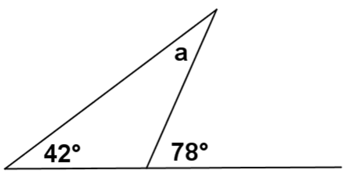
Exercises 1–11

1. Find the measures of angles a and b in the figure to the right. Justify your results.



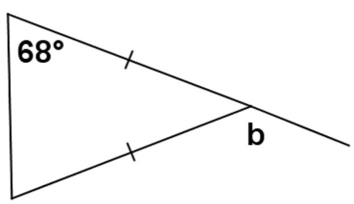
In each figure, determine the measures of the unknown (labeled) angles. Give reasons for your calculations.

2.



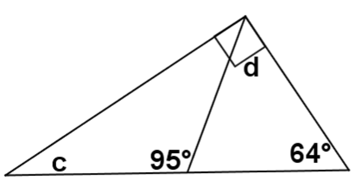
$m\angle a =$ ____________________

3.



$m\angle b =$ ____________________

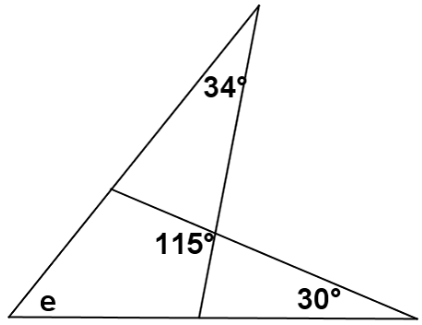
4.



$m\angle c =$ ____________________

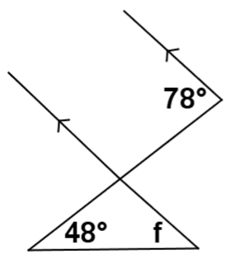
$m\angle d =$ ____________________

5.



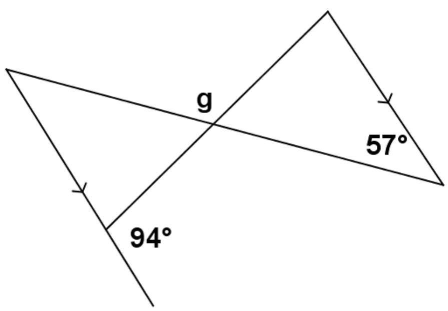
$m\angle e =$ ________________

6.



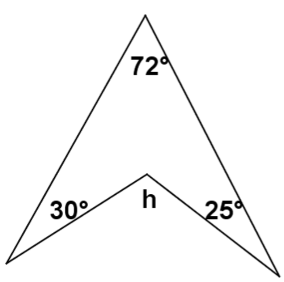
$m\angle f =$ ________________

7.



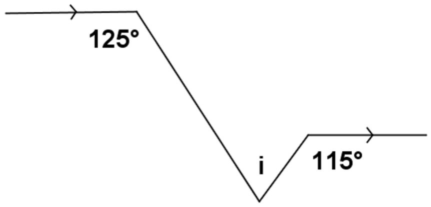
$m\angle g =$ ________________

8.



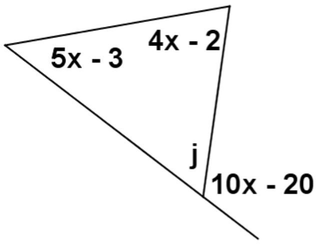
$m\angle h =$ ________________

9.



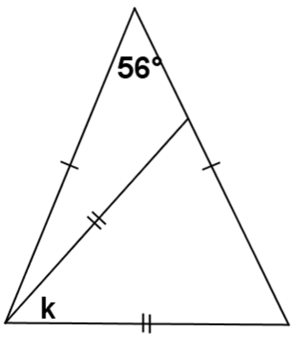
$m\angle i =$ ______________

10.



$m\angle j =$ ______________

11.

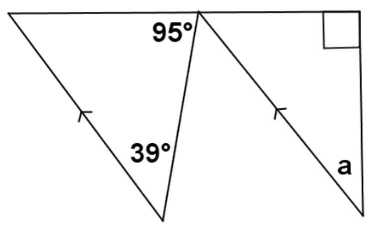


$m\angle k =$ ______________

Problem Set

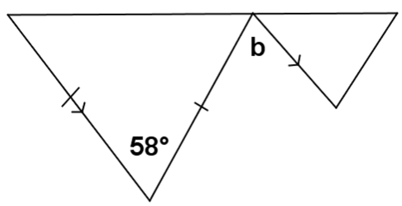
Find the unknown (labeled) angle in each figure. Justify your calculations.

1.



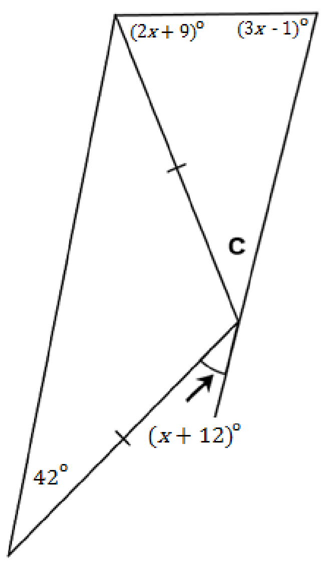
$m\angle a =$ ________________

2.



$m\angle b =$ ________________

3.



$m\angle c =$ ________________

Lesson 9: Unknown Angle Proofs—Writing Proofs

Classwork

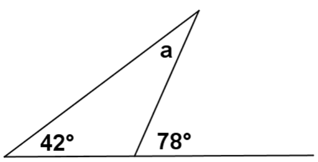
Opening Exercise

One of the main goals in studying geometry is to develop your ability to reason critically, to draw valid conclusions based upon observations and proven facts. Master detectives do this sort of thing all the time. Take a look as Sherlock Holmes uses seemingly insignificant observations to draw amazing conclusions.

Could you follow Sherlock Holmes's reasoning as he described his thought process?

Discussion

In geometry, we follow a similar deductive thought process (much like Holmes uses) to prove geometric claims. Let's revisit an old friend—solving for unknown angles. Remember this one?

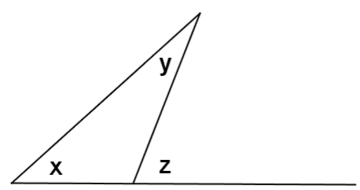


You needed to figure out the measure of a and used the "fact" that an exterior angle of a triangle equals the sum of the measures of the opposite interior angles. The measure of $\angle a$ must, therefore, be $36°$.

Suppose that we rearrange the diagram just a little bit.

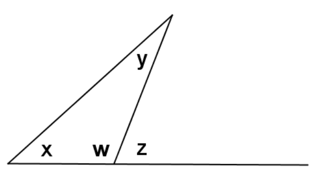
Instead of using numbers, we use variables to represent angle measures.

Suppose further that we already know that the angles of a triangle sum to $180°$. Given the labeled diagram to the right, can we prove that $x + y = z$ (or, in other words, that the exterior angle of a triangle equals the sum of the measures of the opposite interior angles)?



PROOF:

Label $\angle w$, as shown in the diagram.



$m\angle x + m\angle y + m\angle w = 180°$	The sum of the angle measures in a triangle is $180°$.
$m\angle w + m\angle z = 180°$	Linear pairs form supplementary angles.
$m\angle x + m\angle y + m\angle w = m\angle w + m\angle z$	Substitution property of equality
$\therefore m\angle x + m\angle y = m\angle z$	Subtraction property of equality